AN ABSTRACT THEORY OF SENSOR *EVENTIFICATION*

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Overview

Event cameras measure intensity changes and report differences. What's necessary for other sensors to admit eventified versions which provide adequate information despite outputting changes?

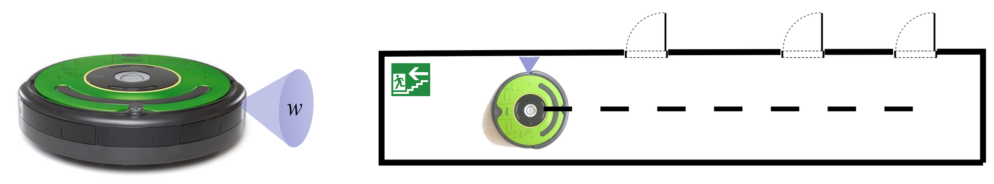
Q: For any sensor of type $X \in$ {compasses, LiDARs, IMUs, …}, is there a useful "event X" version?

This work contributes theory and algorithms (plus a hardness result) along with elementary robot examples.

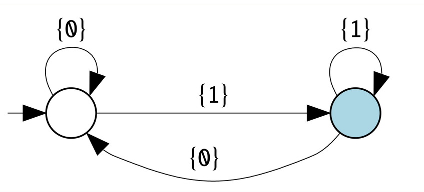
The answer depends upon the signal space and its structure:
- the interplay of the robot and its environment,
- the input–output computation needed to achieve its task,
- access mode: synchronous, asynchronous, polled, triggered.

Example 1 — Space of changes

An iRobot Create drives down a corridor its wall sensor w generating output values as it proceeds. As it does this, the infrared wall sensor on its port side generates a series of binary readings.



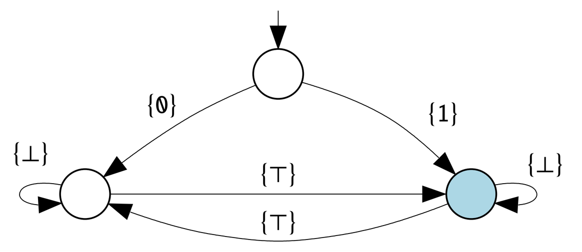
A direct transducer passing through the signal detected has this form:



A sensori-computational device F_{wall}, with $Y(F_{\text{wall}}) = \{0, 1\}$; within the vertices, white encodes $\{0\}$, and azure $\{1\}$.

For a space of 'changes', we introduce a set, $D_2 = \{\bot, \top\}$, and ternary relation written in the form of a table as (row, entry, column) $\in \mathrm{S}_{D_2} \subseteq \{0,1\} \times D_2 \times \{0,1\}$.

	0	1
0	$\bot$	$\top$
1	$\top$	$\bot$



A small device that is F_{wall}'s derivative, *viz.* it is capable of output simulating F_{wall} modulo the delta relation $\nabla\big|^{D_2}_{F_{\text{wall}}}$ defined for D_2.

Delta relation

Definition 1 (delta relation). For relation $\mathrm{S}_D \subseteq Y \times D \times Y$, and device F, the associated *delta relation*, $\nabla\big|^D_F \subseteq \mathcal{L}(F) \times (\{\epsilon\} \cup (Y(F) \cdot D^*))$, is :

0) $\epsilon \nabla\big|^D_F \epsilon$, and

1) $y_0 \nabla\big|^D_F y_0$, for all $y_0 \in Y(F) \cap \mathcal{L}(F)$, and

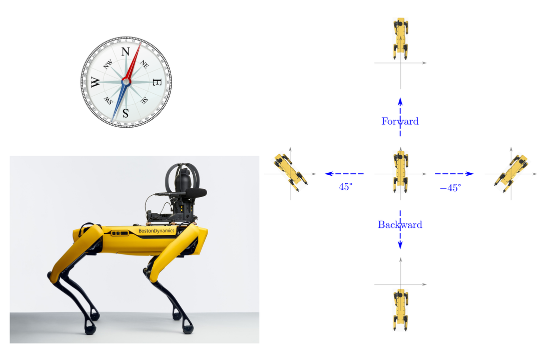
2) $y_0 y_1 \dots y_m \nabla\big|^D_F y_0 d_1 d_2 \dots d_m$, where $(y_{k-1}, d_k, y_k) \in \mathrm{S}_D$.

Question 1. For any sensori-computational device F with (D, S_D), is it $\nabla\big|^D_F$-simulatable?

Revisiting Ex. 1 — Compositionality

Recall Example 1's iRobot Create. These robots also have left and right bump sensors. As these are both binary streams, just like the wall sensor, they can each be transformed with $(D_2 = \{\bot, \top\}, \mathrm{S}_{D_2})$ that tracks bit flips. And to track both, one constructs $\{\bot, \top\} \times \{\bot, \top\}$. In this way, a $d_i = (\bot, \bot)$ would indicate that neither bump sensor's state has changed since previously.

Example 2 — Action-related signals



A Boston Dynamics Minispot quadraped is equipped with a compass to give its heading. To simplify control, it is equipped with motion primitives that, when activated, execute a gait cycle allowing it to move forward a step, move backward a step, or turn in place $\pm 45°$, without losing its footing. Starting facing North, after each motion primitive terminates, the Minispot's heading will be one of 8 directions (the 4 cardinal plus 4 intercardinal ones).

If the raw measurements are $x \in \{\uparrow, \nearrow, \rightarrow, \searrow, \downarrow, \swarrow, \leftarrow, \nwarrow\}$, then an useful variator might employ $D_3 = \{-, \varnothing, +\}$.

Revisiting Ex. 2 — Polling via pump

Reconsider the $45°$ Minispot with a derivative compass for changes $D_3 = \{-, \varnothing, +\}$. Suppose that whenever a downstream consumer of the change-in-bearing information queries, an element of D_3 is produced. If it polls fast enough, we expect that it would contain a large number of $\varnothing$ values. Doubling the rate would (roughly) double the quantity of $\varnothing$ values. At high frequencies, there would be long sequences of $\varnothing$s and those computations on the input stream that are invariant to the rate of sampling would be $\mathrm{P}_{\{\varnothing\}}$-simulatable.

Pump relation

Definition 2 (pump relation). Given $L \subseteq \Sigma^*$ and $\mathrm{N} \subseteq \Sigma$, then the N-*pump* is the relation $\mathrm{P}_{\mathrm{N}} \subseteq L \times \Sigma^*$ defined as follows: $\forall (s_1 \dots s_m) \in L$,

0) $\epsilon \, \mathrm{P}_{\mathrm{N}} \, \epsilon$,

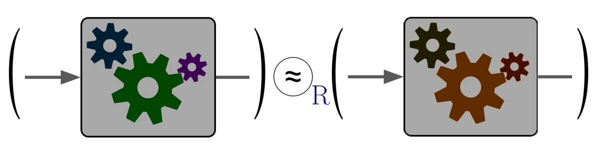
1) $(s_1 \dots s_m) \, \mathrm{P}_{\mathrm{N}} \, (s_1 \dots s_m)$,

2) $\forall b \in \mathrm{N}, k \in \{1, \dots, \ell\}$,
$(s_1 \dots s_m) \mathrm{P}_{\mathrm{N}} (t_1 \dots t_\ell) \implies (s_1 \dots s_m) \mathrm{P}_{\mathrm{N}} (t_1 \dots t_k \, b \, t_{k+1} \dots t_\ell)$.

Question 2. For device F and $\mathrm{N} \subseteq Y(F)$, is F P_{N}-simulatable?

Main ideas

The sensor and basic information processing form a unit: a sensori-computational device with finitely many states. The core notion of behavioral equality is through the lens of a relation.



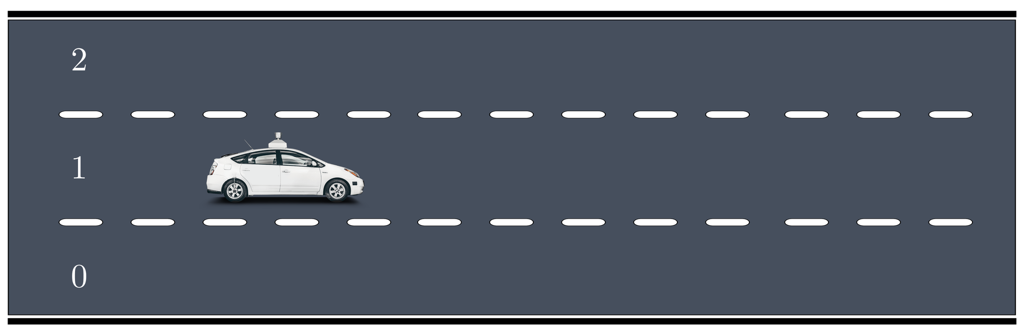
Definition 3 (output simulation modulo a binary relation). Given sensori-computational devices F and F', and relation $\mathrm{R} \subseteq A \times B$ we say that F' *output simulates* F *modulo* R if $\forall s \in \mathcal{L}(F)$:

1. $\exists t_0 \in \mathcal{L}(F')$ such that $s \, \mathrm{R} \, t_0$;
2. $\forall t \in B$ such that $s \, \mathrm{R} \, t$, $t \in \mathcal{L}(F')$ and $C(F, s) \supseteq C(F', t)$.

(Notice that, as $t_0 \in \mathcal{L}(F')$, $C(F', t_0) \neq \emptyset$, hence $C(F, s) \neq \emptyset$.)

Example 4: Arithmetic changes

A self-driving car, shown below, moves on a highway and is equipped with on-board sensors to detect the vehicle's current lane $i \in \{0, 1, 2\}$.



To construct a sensor reporting a change in lane, consider the space $D_{3\text{-lane}} = \{\,\texttt{LEFT}, \texttt{NULL}, \texttt{RIGHT}\,\}$ and, treat the following function as a ternary relation: $\mathrm{S}_{D_{3\text{-lane}}}(i, d) = \min(\max(i + v(d)), 0), 2)$, where $v(\texttt{LEFT}) = +1$, $v(\texttt{NULL}) = 0$, and $v(\texttt{RIGHT}) = -1$.

Then, this transforms any sequence of lane occupancies into unique lane-change signals in a 3-lane road.

Revisiting Ex. 4 — Nonexistence of a useful monoid

Example 4 introduced $D_{3\text{-lane}} = \{\,\texttt{LEFT}, \texttt{NULL}, \texttt{RIGHT}\,\}$. As there are 3 lanes, one might wish to combine, say, two RIGHT actions, one after the other. With only three elements, two RIGHT actions might map to a RIGHT (as that seems less wrong than LEFT or NULL), giving:

$\oplus_1$	LEFT	NULL	RIGHT
LEFT	LEFT	LEFT	NULL
NULL	LEFT	NULL	RIGHT
RIGHT	NULL	RIGHT	RIGHT

But $\oplus_1$ fails to be a monoid operator as since associativity is violated:

$$(\,\texttt{LEFT} \oplus_1 \texttt{LEFT}\,) \oplus_1 \texttt{RIGHT} \neq \texttt{LEFT} \oplus_1 (\,\texttt{LEFT} \oplus_1 \texttt{RIGHT}\,).$$

An alternative that does yield a valid operator, although it is still hard to give it a consistent interpretation:

$\oplus_2$	LEFT	NULL	RIGHT
LEFT	RIGHT	LEFT	NULL
NULL	LEFT	NULL	RIGHT
RIGHT	NULL	RIGHT	LEFT

Now the right action on Y causes difficulty. While NULL must map 0, 1 and 2, each to themselves, the form of $\oplus_2$ requires that the action treat RIGHT $\oplus_2$ RIGHT identically with LEFT. This fails to describe lanes 0, 1 and 2, in a consistent fashion. The lanes do not seem to admit any monoidal variator.

Revisiting Ex. 1, again — Change-triggered sensing via shrink

Example 1's wall sensor produces a stream of 0s and 1s. For change space $D_2 = \{\bot, \top\}$, a derivative exists that produces a stream of $\bot$s and $\top$s, the former occurring when there is no change in the presence/absence of a wall, and latter when there is.

The derivative under the $\{\bot\}$-shrink relation considers whether the desired output can be obtained merely on a sequence of $\top$s. If the output depends on a count of $\top$s, (like even- *vs* odd-numbered doorways), then this is possible. If it depends on a count of $\bot$s, or the interleaving of $\top$s and $\bot$s then it can not.

If some derivative, F' say, is $\pi_{\{\bot\}}$-simulatable, then it can operate effectively even if it is notified only when the wall-presence condition changes. In this sense that such F's are change-triggered.

Example 3 — Additional action-related signals

Suppose Example 2's Minispot is enhanced so its motion library includes primitive allowing it to turn in place by $\pm 90°$. Now, after each motion terminates, the compass signal can include changes for which D_3 is inadequate. When Definition 1 is followed to define $\nabla\big|^{D_3}_F$, those sequences involving $90°$ changes fail to find any $d_k \in D_3$, and the relation is not left-total $\implies$ there can be no derivative.

A more sophisticated choice *does* allow a derivative.

Revisiting Ex. 3 — Monoidal structure

In Example 3, suppose we encode $\{\uparrow, \nearrow, \dots \nwarrow\}$ with headings as integers $\{0, 45, \dots, 315\}$. For the triple relation take the usual addition on integers, $+ : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, but restricted so the first and third slots only have elements within $\{0, 45, \dots, 315\}$. This is, $\mathbb{Z}_8$, the cyclic group of order eight.

Shrink relation

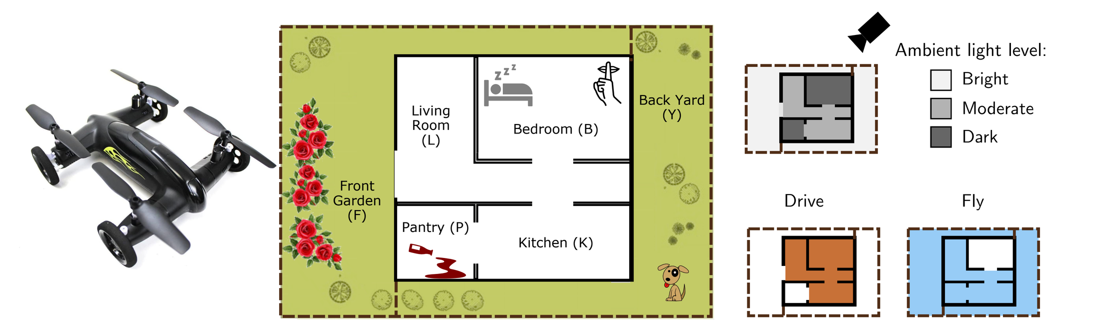
Definition 4 (shrink relation). Given $L \subseteq \Sigma^*$ and $\mathrm{N} \subseteq \Sigma$, the N-*shrink* is the single-valued, total function π_{N} defined recursively as follows:

$$\begin{aligned} \pi_{\mathrm{N}} : L &\to (\Sigma \setminus \mathrm{N})^*, \\ \epsilon &\mapsto \epsilon, \\ s_1 \dots s_m &\mapsto s_1 \dots s_m \text{ if } \forall j \in \{1, \dots, m\}, s_j \notin \mathrm{N}, \\ s_1 \dots s_i \dots s_m &\mapsto \pi_{\mathrm{N}}(s_1 \dots s_{i-1} s_{i+1} \dots s_m) \text{ when } s_i \in \mathrm{N}. \end{aligned}$$

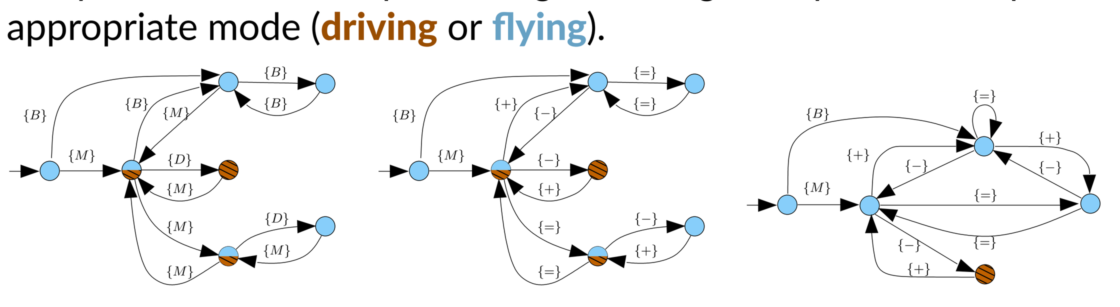
Question 3. For device F and $\mathrm{N} \subseteq Y(F)$, is F π_{N}-simulatable?

Example 5 — Stability and chatter-free behavior

A Syma X9 Quadcopter Car, capable of both flight and wheeled locomotion, monitors a home environment. The robot is equipped with a single-pixel camera. It must fly to avoid grass outdoors (F and Y) and liquids in the pantry (P); in the bedroom (B), it should drive to minimize noise.



The robot is initially located in either the front garden (F) or the living room (L). To determine its state, the robot uses its single-pixel camera, which is capable of discerning just three different ambient light levels (Bright (B), Moderate (M), Dark (D)). The following four sensori-computational devices process light readings as input, and output the appropriate mode (**driving** or **flying**).



(a) (b) (c)

Here, device (a) works in the original signal space, it is a state diagram that essentially transcribes the problem, serving as a specification for acceptable input–output functionality.

The other two, (b) and (c), are derivatives that operate in the space of changes $D_\ell = \{+, -, =\}$, which uses $+$ to capture the brightness increases, $-$ for brightness decreases $=$ for brightness equivalence. Device (c) not only chooses a single output for each vertex but is output stable.

Shrink and Pump

Theorem. (equivalence of pumping and shrinking) For device F and $\mathrm{N} \subseteq Y(F)$, such that all $s_1 s_2 s_3 \dots s_k \in \mathcal{L}(F)$ have $s_1 \in Y(F) \setminus \mathrm{N}$:

$$F \text{ is } \mathrm{P}_{\mathrm{N}}\text{-simulatable} \iff F \text{ is } \pi_{\mathrm{N}}\text{-simulatable}$$

Monoidal variator and disaggregator relation

Definition 5 (monoidal variator). A *monoidal variator* for observation set Y is a monoid $(D, \oplus, 1_D)$ and a right action of D on Y, $\bullet : Y \times D \to Y$. That is $\bullet$ is a total function with:

identity: $y \bullet 1_D = y$;

compatibility: $(y \bullet d_1) \bullet d_2 = y \bullet (d_1 \oplus d_2)$, $\forall y \in Y$, and all d_1, d_2 in D.

Definition 6 (monoid disaggregator). Given the monoid $(D, \oplus, 1_D)$ and observation set Y, the associated *monoid disaggregator* is a relation, $\partial\oplus\big|_Y \subseteq (\{\epsilon\} \cup (Y \cdot D^*)) \times (\{\epsilon\} \cup (Y \cdot D^*))$ defined as:

0) $\epsilon \, \partial\oplus\big|_Y \epsilon$, and

1) $y_0 \, \partial\oplus\big|_Y y_0$ for all $y_0 \in Y$, and

2) $y_0 d_1 d_2 \dots d_m \, \partial\oplus\big|_Y y_0 d'_1 d'_2 \dots d'_n$ if $d_1 \oplus d_2 \oplus \dots \oplus d_m = d'_1 \oplus d'_2 \oplus \dots \oplus d'_n$.

Question 4. For device F and with monoidal variator $((D, \oplus, 1_D), \bullet)$ on $Y(F)$, is it $\left(\nabla\big|^D_F \,\mathbin{⨾}\, \partial\oplus\big|_{Y(F)}\right)$-simulatable?

Example 6 — Robot with irrecoverable error

The delivery robot above has a sensor to detect some irreversible condition. Once triggered, the sensor retains this status permanently. Representing the robot's status by 0 for 'normal' and 1 for 'abnormal', we may then use a monoid variator $D_2 = \{☺, ☹\}$, with 1_{D_2} is ☺, and the monoid operator $\oplus$ and the right action $\bullet$ defined here:

$\oplus$	☺	☹
☺	☺	☹
☹	☹	☹

and

$\bullet$	☺	☹
0	0	1
1	1	1

Bridging via the monoid integrator

Definition 7 (monoid integrator). For observations Y and monoid $(D, \oplus, 1_D)$, the associated *monoid integrator* is a function $\int\!\oplus\big|_Y$ is:

$$\begin{aligned} \textstyle\int\!\oplus\big|_Y : \{\epsilon\} \cup (Y \cdot D^*) &\to \{\epsilon\} \cup Y \cup (Y \cdot D) \\ \epsilon &\mapsto \epsilon \\ y_0 &\mapsto y_0 \\ y_0 d_1 d_2 \dots d_m &\mapsto y_0 (d_1 \oplus d_2 \oplus \dots \oplus d_m). \end{aligned}$$

Theorem. For any device F with monoidal variator $(D, \bullet)$, $1_D \notin Y(F)$, which is $\left(\nabla\big|^D_F \,\mathbin{⨾}\, \int\!\oplus\big|_F\right)$-simulatable, there exists a single F' such that:

1) $F' \sim F \left(\bmod \nabla\big|^D_F \,\mathbin{⨾}\, \partial\oplus\big|_{Y(F)}\right)$,
2) $F' \sim F \left(\bmod \nabla\big|^D_F \,\mathbin{⨾}\, \mathrm{P}_{\{1_D\}}\right)$,
3) $F' \sim F \left(\bmod \nabla\big|^D_F \,\mathbin{⨾}\, \pi_{\{1_D\}}\right)$,
4) F' is vertex stable with respect to $\{1_D\}$, and
5) F' is output stable with respect to $\{1_D\}$.