Abstract—Scores of papers show, given some robots, how to improve the useful work they perform. Continuing this line, we consider the efficiency of robot experiments by examining the feasibility of conducting several experiments simultaneously, interleaving execution and sharing resources between them. This paper lays theoretical groundwork for that concept and demonstrates its feasibility and utility.

I. INTRODUCTION

Demonstrations of capabilities or claims on robot hardware are more persuasive than the same in pure simulation. But robot hardware is burdensome: it is expensive to build or purchase; considerable time must be expended in maintenance, repair, and also operation. Multiple robot systems are only more so. Thankfully, the community has learned to amortize some of these costs through re-use and sharing: designs, schematics, and configurations for several platforms have been released and made available so that others may benefit [1–5]. Increasingly, we see not only hardware platforms, but broader test-beds [6–12]. Several existing robot test-beds directly facilitate experiments, culminating in shared, remotely-accessible scientific infrastructure [13, 14]. But we ask: to extract greater use from such resources, can re-use and sharing go further still, be finer-grained and more dynamic?

Decades of computer systems research and technology development have shown how to boost the availability, utilization, and effectiveness of computing devices through multiprogramming, multitasking, and multiprocessing (via multithreading and multicore architectures). Notwithstanding that some later instances grew out of earlier ones, this multitude of multis can all be understood as being flexible in how resources are employed. We contend that a similar approach might be used for robots, with the ultimate aim of improving availability, utilization, and effectiveness of robot hardware. In this paper we are specifically interested in common, general, re-usable robot infrastructure designed to conduct robot hardware experiments [13]. For already existing infrastructure, to maximize their benefit, idle portions may well be put to productive use. The idea explored in this paper, and illustrated in Figure 1, is to leverage hardware resources more effectively by intermingling the execution of more than one robot experiment on the same test-bed.

The question of what constitutes the essence of a valid robot experiment is a complex, interesting, and even philosophical one. For the present paper the core consideration is how, without losing that essence, to provide an opening for the flexible apportioning of resources. We provide an answer: experiments produce a stream of sensor readings contingent on states visited and influenced via selection of actions by robots, such selections being made in light of earlier sensor readings. This point of view, where sensing may be manipulated up the perceptual limits of the agents, has been suggested before in robotics (see [15, 16]), and is now widely employed in biology (see the entire special-issue in [17]). Formalizing this sensing-oriented point of view provides the scaffold over which we build a conception of multi-experiments. Beyond the degrees-of-freedom of the robots in the test-bed itself, this context identifies specific additional freedoms which can be used to optimize execution of collections of experiments: freedom to warp time, freedom to distort robot identity.

But how this might bear fruit, concretely? Suppose a roboticist has developed some control software for their favorite mobile robot, say, a controller based on artificial potential fields [18]. This controller has several parameters which manipulate gains and weight various competing factors. Usually these are understood to be empirical parameters, items tuned by running the robot and making adjustments interactively. When some criterion can be provided (such as straightness of travel subject to exemption from collisions) then the time to arrive at a set of suitable parameters depends on completing sufficient runs across the ranges of parameters. If multiple runs can occur in quick succession, or even simultaneously, then overall productivity will be increased.
This is precisely what multi-experiments enable.

The contribution of this paper is to lay a foundation for understanding this notion of multi-experiments, including introducing some fundamental definitions (§ II), establishing conditions on when multi-experiments may be achieved (§ III), examining differences from standard notions of multi-processing (§ IV), and demonstrating a simple but practical multi-experiment (§ V).

II. DEFINITIONS

This section introduces the basic definitions from which the subsequent analysis proceeds, starting with this definition of an idealized system in which multiple robots operate.

Definition 1. A deterministic multi-robot transition system [16] is a 8-tuple \((n, X, U, f, d, Y, h, x_0)\), in which

1) \(n\) is a positive integer identifying the number of robots,
2) \(X = X^{(1)} \times \cdots \times X^{(n)}\) denotes a state space, composed of individual state spaces for each robot,
3) \(U = U^{(1)} \times \cdots \times U^{(n)}\) denotes an action space, composed of individual action spaces for each robot,
4) \(f : X \times U \rightarrow X\) is a state transition function, defined in terms of transition functions \(f^{(1)}, \ldots, f^{(n)}\) for each robot, so that
   \[ f((x^{(1)}, \ldots, x^{(n)}), (u^{(1)}, \ldots, u^{(n)})) = (f^{(1)}(x, u^{(1)}), \ldots, f^{(n)}(x, u^{(n)})), \]
5) \(d : X \times U \rightarrow [0, \infty)\) is a transition duration function, indicating the amount of time that elapses when executing each action from each state,
6) \(Y = Y^{(1)} \times \cdots \times Y^{(n)}\) denotes an observation space, composed of individual observation spaces,
7) \(h : X \rightarrow Y\) is an observation function, defined in terms of observation functions \(h^{(1)}, \ldots, h^{(n)}\) for each robot, so that \(h(x) = (h^{(1)}(x), \ldots, h^{(n)}(x))\),
8) \(x_0 \in X\) is the system’s initial state.

The execution of such a system proceeds in discrete stages, indexed \(k = 1, 2, \ldots\), through a sequence of states \(x_0, x_1, \ldots\), influenced by the actions \(u_0, u_1, \ldots\) selected by each robot.

Example 1. A straightforward example, first introduced in a prior paper [16], illustrates the concept. See Fig. 2. Suppose \(n\) robots move along a single-lane road. Each robot \(r_i\) controls its own speed, within an allowable range \([v_{\text{min}}, v_{\text{max}}]\). Sensors enable each robot to detect the distance to its immediate neighbors both ahead and behind. This scenario is readily modeled as a deterministic multi-robot transition system

\[ S_{n, v_{\text{min}}, v_{\text{max}}}(\mathbb{R}^n, [v_{\text{min}}, v_{\text{max}}], f, d, \mathbb{R}^n \times \mathbb{R}^+, h, x_0), \]

in which each state \(x \in \mathbb{R}^n\) represents the position along the road of each of the robots, actions represent each robot’s chosen velocity at a given time step, and each observation encodes the distance to the nearest robot behind and ahead. We refer the reader to the original paper [16] for more detail.

Example 2. We note concisely here that public-facing robot multi-robot test-beds such as the Georgia Tech Robotarium [13] can be modeled as deterministic multi-robot transition systems, using the appropriate models of transitions and observations. We used this platform to conduct a number of proof-of-concept executions of the strategies described in this paper—see Figure 3—and utilized a simulator mimicking it to obtain the quantitative results that appear below.

A. (Multi-)illusions

Our primary interest is in the ability of one deterministic multi-robot transition system to present, to some of its robots, the appearance of operating within another system. Particularly relevant is the case in which such illusions are maintained for multiple systems within the same host. The next definition formalizes this idea, generalizing an earlier formulation [16] for single systems.

Definition 2. For some finite set of deterministic multi-robot transition systems \(S = S_1, \ldots, S_d\) where for each \(1 \leq j \leq d\), \(S_j = (n_j, X_j, U_j, f_j, h_j, x_0)\) and \(\hat{S} = (\hat{n}, \hat{X}, \hat{U}, f, \hat{Y}, \hat{h}, \hat{x}_0)\) and a positive integer \(m_j\), we say that \(\hat{S}\) presents a multi-illusion for \(S\) if there exist

(i) robot policies \(\hat{\pi}^{(1)}, \ldots, \hat{\pi}^{(\hat{n})}\) in \(\hat{S}\),
(ii) a strictly increasing function \(z_j : \mathbb{Z}_+ \rightarrow \mathbb{Z}_+\), and
(iii) an infinite series of functions \(\rho_{j,k} : \mathbb{Z}_{m_j} \rightarrow \mathbb{Z}_{\hat{n}}\)

for any robot policies \(\pi_j^{(1)}, \ldots, \pi_j^{(n_j)}\) in each \(S_j\), such that for all \(k \geq 0\) and all \(1 \leq i \leq m_j\), we have

\[ h_j^{(i)}(x_{j,k}) = \hat{h}^{(\rho_{j,k}(i))}(\hat{z}_j^{(k)}). \quad (\ast) \]

We refer to the specific choices of policies and functions satisfying this constraint as a realization of the illusion.

Here each of the robots’ policies \(\hat{\pi}^{(1)}, \ldots, \hat{\pi}^{(\hat{n})}\) in \(\hat{S}\) is a function mapping from information available to that robot.
Example 3. Recall the caravan systems from Example 1. For any two such systems $S_1 = S_{\text{min}, v_{\text{min}}, v_{\text{max}}}$ and $S_2 = S_{\text{min}, v_{\text{min}}', v_{\text{max}}'}$, we can form a multi-illusion of the two systems, with $m_1 = m_2 = 1$, within a host system of the form $\tilde{S} = S_{\text{min}, \tilde{v}_{\text{min}}, \tilde{v}_{\text{max}}}$.

One way to construct this illusion is to select policies $\tilde{\pi}$ in which robot 1 moves at a constant speed $(\tilde{v}_{\text{min}} + \tilde{v}_{\text{max}})/2$. Robots 2 and 3, knowing the desired observation $y_{1k} = (\alpha_{1k}^{(1)}, \beta_{1k}^{(1)})$ from $S_1$, position themselves on opposite sides of robot 1, moving as fast as possible at each stage in $\tilde{S}$ toward positions where $\tilde{x}_{1k}^{(1)} - \tilde{x}_{1k}^{(2)} = b_{1k}^{(1)}$ and $\tilde{x}_{1k}^{(3)} - \tilde{x}_{1k}^{(1)} = a_{1k}^{(1)}$.

Then robots 4 and 5, knowing the desired observation $y_{2k} = (\alpha_{2k}^{(1)}, \beta_{2k}^{(1)})$ from $S_2$, position themselves to the right of robot 3 such that $\tilde{x}_{2k}^{(4)} - \tilde{x}_{2k}^{(3)} = b_{2k}^{(1)}$ and $\tilde{x}_{2k}^{(5)} - \tilde{x}_{2k}^{(4)} = a_{2k}^{(1)}$. To satisfy the remaining conditions of Definition 2, we define $z_1$ to return the time when robots 2 and 3 in $\tilde{S}$ have reached their target positions. Similarly, $z_2$ returns the time when robots 4 and 5 in $\tilde{S}$ have reached their target positions. Figure 4 illustrates the construction, which can, of course, also be generalized to present a multi-illusion of any number $d$ of $S_1, \ldots, S_d$ systems within $\tilde{S} = S_{2d+1, v_{\text{min}}, \tilde{v}_{\text{min}}}$.

But that same effect might also be achieved by holding the robots playing the role of obstacles fixed in place, and allowing the recipient robot to move normally. Upon nearing the boundary of the available workspace, the illusion can ‘reset’ to the center, utilizing a suitable $z$ function to halt the execution until the shift back to center is complete. Figure 5 depicts the relative progress of these two illusions as time proceeds.

Example 4. Significant differences in the realization of a multi-illusion may be realized even in the basic case in which $d = 1$. For example, prior results [16, Example 5] demonstrated that a Robotarium-like system can present an illusion of a mobile robot traversing an arbitrarily large field of disc-shaped obstacles, by holding the recipient robot fixed at the center of the workspace and positioning the other robots at appropriate locations relative to that center, based on the locations of obstacles visible to the recipient robot.
For a given deterministic multi-robot transition system \((n, X, U, f, d, Y, h, x_0)\) and equivalence relation on observations \(\equiv \subseteq Y \times Y\), a relation on its states \(\sim \subseteq X \times X\) is an observation-simulation relation if \(\sim\) is an equivalence relation, and for any \(x_1, x_2 \in X\) for which \(x_1 \sim x_2\), we have

1) \(h(x_1) \equiv h(x_2)\), and
2) \(\forall u_1 \in U, \exists u_2 \in U\) such that \(f(x_1, u_1) \sim f(x_2, u_2)\).

When the observation equivalence \(\equiv\) is taken as equality, an observation-simulation equivalence relation has much of the character of a bisimulation relation [19]. Definition 5 generalizes the bisimulation notion, which would have \(u_2 = u_1\) in the second condition.

Notice that, given an observation-simulation \(\sim_a\) using \(\equiv_a\) on \(Y\), if \(\equiv_b\) is a finer relation on observations than \(\equiv_a\), then \(\sim_a\) is an observation-simulation using \(\equiv_b\) as well.

**Definition 6.** For a given deterministic multi-robot transition system \((n, X, U, f, d, Y, h, x_0)\), an observation-simulation relation \(\sim \subseteq X \times X\), the quotient graph is a directed graph whose vertex set is the set of equivalence classes of \(\sim\). Each such class can be represented by an arbitrary representative: \([x] := \{x' \in X \mid x \sim x'\}\). Between any two vertices \([x_1]\) and \([x_2]\), an edge \([x_1] \rightarrow [x_2]\) exists if and only if there exists some action \(u \in U\) for which \([x_2] = [f(x_1, u)]\).

Note that the quotient graph may not necessarily be a finite graph (if the state space \(X\) is infinite) nor even locally finite (if the action space \(U\) is infinite).

**Example 6.** Referring again to Example 1, consider the system \(S_{n,v_{\min},v_{\max}}\). Take \(\equiv\) to be the identity relation and define \(\sim\) such that for any pair of states \(x_1\) and \(x_2\) we have \(x_1 \sim x_2\) if and only if \(h(x_1) = h(x_2)\), which occurs precisely when the inter-vehicle spacing is identical between the two states, regardless of those states’ absolute positions along the roadway. Notice that the construction does indeed produce an observation-simulation relation. Moreover, the quotient graph in this case is strongly connected, since the robots can, in some finite series of stages, adjust positions relative to each other arbitrarily. Note in particular that this remains true even if \(v_{\min} > 0\) — because \(\sim\) ensures that only the relative positions of the robots are relevant, paths exist between any pair of equivalence classes, without the need for any robot to move backward in absolute position.

**A. Time-sliced multi-illusions**

We can now use the idea of the quotient graph to give conditions under which a sort of ‘time slicing’ illusion can be guaranteed to exist.

**Theorem 1.** Let \(S_1\), \(S_2\), and \(\hat{S}\) denote three systems, and suppose there exist illusions of \(S_1\) by \(\hat{S}\) and of \(S_2\) by \(\hat{S}\). If there exists an observation-simulation relation for \(\hat{S}\) for which the quotient graph is strongly connected, then there exists a multi-illusion of both \(S_1\) and \(S_2\) by \(\hat{S}\).

**Proof roadmap:** Under the presumed conditions, portions of the execution of the individual illusions for \(S_1\) and \(S_2\) may be interleaved. Let \(\tau \in \mathbb{Z}^+\) denote an arbitrary positive time slice, to be interpreted as a number of stages elapsed in \(\hat{S}\). We can construct a realization of the multi-illusion for both \(S_1\) and \(S_2\) that cycles through four phases: (1) an execution of the illusion for \(S_1\) for \(\tau\) stages, (2) an interregnum of actions that transition to a state suitable to begin or continue the \(S_2\) illusion, (3) an execution of the illusion for \(S_2\) for \(\tau\) stages, and (4) an interregnum of actions that transition to a state suitable to continue the \(S_1\) illusion. The key observation is that the strong connectedness of the quotient graph ensures action sequences always exist that can achieve phases (2) and (4).

**Example 7.** Figure 7 shows a progress plot for an example in which the construction for Theorem 1 is utilized to time share two of the sorts of illusions introduced in Example 4 within a single simulated Robotarium.
Corollary 1. Under the same premises as Theorem 1, suppose the construction in its proof is used to provide a multi-illusion for \( S = \{S_1, S_2\} \) by \( \hat{S} \). For \( j \in \{1, 2\} \), let \( L_j^S(\hat{t}_k) \) denote the latency for \( S_j \) in this combined illusion. Then
\[
L_j^S(\hat{t}_k) \leq \frac{\hat{t}_k}{\beta(k)} \cdot L_j(\hat{t}_{\beta(k)}),
\]
in which \( \beta(k) = \tau \left\lfloor \frac{\hat{t}_k}{2(\tau + \text{diam}(G))} \right\rfloor \).

Proof roadmap: Observe that after \( \hat{k} \) stages, the number complete cycles of the four phases of the constructed illusion is at least \( \left\lfloor \frac{\hat{k}}{2(\tau + \text{diam}(G))} \right\rfloor \), since each instance of phases (1) or (3) consumes precisely \( \tau \) stages, and each instance of phases (2) or (4) consumes at most \( \text{diam}(G) \) stages. The illusion for \( S_j \) executes for \( \tau \) stages in each of these cycles, amounting to \( \tau \left\lfloor \frac{\hat{k}}{2(\tau + \text{diam}(G))} \right\rfloor \) stages in total for \( S_j \). Let \( k_j^m = \max\{k \in \mathbb{Z}^+ | \frac{\hat{t}_k}{2(\tau + \text{diam}(G))} \leq \beta(k) \} \) and observe that
\[
L_j^S(\hat{t}_k) = \frac{\hat{t}_k}{t_{k_j^m}} \cdot L_j(\hat{t}_{\beta(k)}),
\]
applying Definition 3 in the final step to complete the proof.

B. Leveraging role reassignments

Moving now beyond mere time slicing, note that in Definition 2, for the (\( \ast \)) equation corresponding to system \( S_j \), the \( i \)-th element may be re-mapped (via the \( \rho_j \) function) to be some arbitrary element observed in \( \hat{S} \). Since there is no requirement that \( \rho_j \) be injective, duplicate values are entirely superfluous. Thus, though prior discussion of specific cases, as in Example 6, has been for an observation-simulation relation \( \sim \) as equality on elements of \( Y \), equality is stronger than is strictly necessary for illusions. This motivates the following function, \( \pi \), which repackages data from an \( n \)-vector into a set:
\[
\left( y^{(1)}, \ldots, y^{(n)} \right) \mapsto \pi \left( \bigcup_{i=1}^{n} y^{(i)} \right).
\]

And, hence, let equivalence relation \( \cong \subseteq Y \times Y \) be defined as \( y \cong y' \) if and only if \( \pi(y) = \pi(y') \).

Using \( \cong \) on \( Y \) for some deterministic multi-robot transition system \( (n, X, U, f, d, Y, h, x_0) \), if we are given some observation-simulation equivalence relation \( \sim \), we can construct the quotient graph \( G_{\sim} \). (In this case we may think of the vertices of the graph as labelled by sets.) Theorem 1 holds for \( \sim \), but observe that the time-slicing used in the construction as part of the proof argument only modifies the \( z_j \) functions; essentially it is purely a strict interleaving the illusions for \( S_1 \) and \( S_2 \). Under \( \cong \), one might do better: rather than reaching a state \( x \in [x]_{\sim} \) where the \( \rho_j \) for the multi-illusion is identical with the original illusion for \( S_j \), instead one reaches \( x' \in [x]_{\cong} \) and uses \( \rho_j' \) constructed to map to some robot producing the required observation (different \( x' \) will have different \( \rho_j' \)).

Further, because the identity relation is finer than \( \cong \), i.e., \( \text{id}_Y \subseteq \cong \), every observation-simulation relation \( \sim \) via the former, is one for \( \cong \) too. We would expect there to be additional observation-simulation relations \( \sim \) for \( \ast \), generally.

C. Multi-illusions via lim \( \sup \)

To give a different, tighter characterization for sufficiency than the preceding, two additional definitions are needed.

Definition 7. Given a multi-robot transition system \( S = (n, X, U, f, d, Y, h, x_0) \), the observation footprint of \( S \) is
\[
F(S) = \bigcup_{k \geq 0} \bigcup_{k \in \mathbb{N}} \{ \pi(h(x_k)) \}.
\]

Intuitively, the observation footprint is a collection all the sets of observations that might be seen by the system at any particular point in time.

Definition 8. A walk on graph \( G = (V, E) \) starting at \( [x_0] \in V \) is a function \( w : \mathbb{Z}^+ \to V \), with \( w(0) = [x_0] \) and such that, for \( n \geq 1 \), there is an edge \( ([x_{n-1}] \to [x_n]) \) in \( E \).

Now, denoting the powerset by \( \mathcal{P}(\cdot) \), we state the theorem.

Theorem 2. There exists a multi-illusion for \( S = S_1, \ldots, S_d \) on \( \hat{S} \), if there exists an observation-simulation relation for \( \hat{S} \), which induces a quotient graph \( G \), and some walk \( w \) on \( G \) exists such that
\[
\bigcup_{j=1}^{d} F(S_j) \subseteq \text{lim sup}_{n \to \infty} \mathcal{P}(\pi(w(n))).
\]

Proof roadmap: Given such a walk \( w \), one can ensure progress is made in any of the \( d \) systems. In \( S_i \), any set of observations you wish to concoct appears in \( F(S_i) \). The condition, thus, ensures that, at any point in time, a set containing it will appear in finite time. Constructing the illusion requires choosing \( z_i \) so it progresses when such a set appears, and having \( \rho_i \) unpack the observations needed and dispatch them to the intended recipients.
used productively for any system(s) essence, the observations obtained by robots in examining what, precisely, accounts for this windfall. In seems perhaps to connote some definitional flaw) it is worth speedup. Thus, by selecting a large $S$ of these copies are executed sequentially. But $S$ speedup between between the two multi-illusions is in Definition 2 is satisfied $d$ times by mere copying. The speedup between between the two multi-illusions is $S = d$. Thus, by selecting a large $d$, we can obtain arbitrarily large speedup.

Since Example 8 has the appearance of chicanery (and seems perhaps to connote some definitional flaw) it is worth examining what, precisely, accounts for this windfall. In essence, the observations obtained by robots in $S$ may be used productively for any system(s) $S$, that make progress after seeing a subset of those observations. Further, because there is indirection between state configurations and observations, a single robot in $S$ may simultaneously service multiple illusions. Indeed, we witness this in practice, as noted below.

V. CASE STUDY: PARAMETER OPTIMIZATION

To bring our analysis full circle back to the promise of efficient experimentation in multi-robot test-beds, this section describes a simple example realization of that idea. To maintain focus on the novel aspects of the setting, let us consider the straightforward and well-understood task of optimizing a parameter in an artificial potential field controller.

Recall that this sort of controller uses parameters $\eta$ and $\zeta$ to weight the forces pushing the robot away from obstacles and toward the goal, respectively. One may desire to conduct an experiment to determine the best values for $\eta$ and $\zeta$ to balance these forces and to achieve the fastest travel time to a goal. In such a scenario, we might hold the sum $\eta + \zeta$ fixed, and conduct a series of trials, varying $\zeta$ and measuring the robot’s travel time for each trial.

We conducted this experiment via multi-illusion in a simulated Robotarium. The specific form of the multi-illusion divides the workspace into left and right halves, and executes a distinct trial on each side. However, the two sides are sufficiently close together that some robots representing obstacles sometimes play different roles at the same time in both ongoing trials. Figure 8 shows an example: Two illusioned robots (#1 highlighted in green, #2 highlighted in purple) are observing what they believe to be an obstacle, as concocted by robot #4 (in the cyan square) for both of them simultaneously. This resource sharing enables the illusions to proceed more rapidly than might have been expected. Figure 9 shows the overall progress of the process for an experiment with 10 trials, executed in simultaneous pairs. Of particular interest there is the high density of simultaneous execution. We also conducted a larger scale version of this experiment, using more finely-grained selection of $\zeta$ values and conducting 10 trials for each value to account for randomness in obstacle placement. Figure 10 shows the results, in which potential field controller parameter $\zeta = 12.5$ gave the best performance.

VI. CONCLUSION

Identifying the concept multi-experiments as a topic of study, this paper lays the initial foundations on which further development should build. It provides definitions and conditions for the existence of multiplexing, as well as results of a more quantitative nature in the form of performance bounds. Further, the paper has presented demonstrations of the feasibility of the concept in practice. Though the implemented examples are simple, they already showcase the utility of the approach, for instance, in parameter tuning. Perhaps most significant is that different multi-illusions can be seen to have vastly different performance characteristics. Further work would need to explore these aspects more completely, potentially offering tighter bounds on the basis of opportunities for optimization, and expand the theory beyond the basic deterministic model.

REFERENCES


