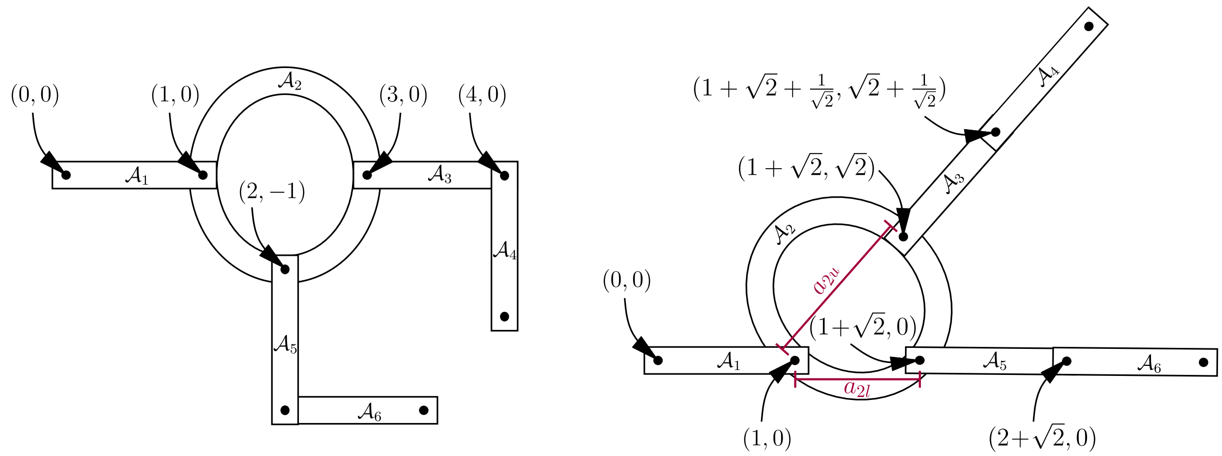

Figure 1: A tree of bodies in which the joints are attached in different places.

Example 3.6 (A 2D Tree of Bodies) Figure 1 shows a 2D example that involves six links. To transform $(x, y) \in \mathcal{A}_6$, the only relevant links are $\mathcal{A}_5$, $\mathcal{A}_2$, and $\mathcal{A}_1$. The chain

$$T_1 T_{2l} T_5 T_6 \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}, \tag{1}$$

in which

$$T_1 = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & x_t \\ \sin\theta_1 & \cos\theta_1 & y_t \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2}$$

$$T_{2l} = \begin{pmatrix} \cos\theta_{2l} & -\sin\theta_{2l} & a_1 \\ \sin\theta_{2l} & \cos\theta_{2l} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & -\sin\theta_2 & 1 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3}$$

$$T_5 = \begin{pmatrix} \cos\theta_5 & -\sin\theta_5 & a_{2l} \\ \sin\theta_5 & \cos\theta_5 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\theta_5 & -\sin\theta_5 & \sqrt{2} \\ \sin\theta_5 & \cos\theta_5 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

and

$$T_6 = \begin{pmatrix} \cos\theta_6 & -\sin\theta_6 & a_5 \\ \sin\theta_6 & \cos\theta_6 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\theta_6 & -\sin\theta_6 & 1 \\ \sin\theta_6 & \cos\theta_6 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

The matrix T_{2l} in (3) denotes the fact that the lower chain was followed. The transformation for points in $\mathcal{A}_4$ is

$$T_1 T_{2u} T_3 T_4 \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}, \tag{6}$$

in which T_1 is the same as in (2), and

$$T_3 = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & a_{2u} \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 2 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{7}$$

and

$$T_4 = \begin{pmatrix} \cos\theta_4 & -\sin\theta_4 & a_3 \\ \sin\theta_4 & \cos\theta_4 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\theta_4 & -\sin\theta_4 & 1 \\ \sin\theta_4 & \cos\theta_4 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{8}$$

The interesting case is

$$T_{2u} = \begin{pmatrix} \cos\theta_{2u} & -\sin\theta_{2u} & a_1 \\ \sin\theta_{2u} & \cos\theta_{2u} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos(\theta_{2l} + \pi/4) & -\sin(\theta_{2l} + \pi/4) & 1 \\ \sin(\theta_{2l} + \pi/4) & \cos(\theta_{2l} + \pi/4) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{9}$$

in which the constraint $\theta_{2u} = \theta_{2l} + \pi/4$ is imposed to enforce the fact that $\mathcal{A}_2$ is a junction.