CSCE625: Artificial Intelligence
Programming Assignment 3: Simplifying Mathematical Expressions via Search

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This is posted the week of February 18, 2022. The intended submission date is March 6th, 2022. Submission will be via upload to canvas.

Problem Domain

We are interested in automating the process of simplifying symbolic mathematics. If you’ve used MATHEMATICA, MATLAB, or MAPLE, chances are that you’ve seen how symbolic mathematics can be manipulated automatically in order to reduce, solve, or simplify algebraic expressions. This assignment asks you to write a program to manipulate symbolic mathematics. Rather than using ad hoc pattern matching, the intention is to have you focus on search as a unifying approach.

Assignment

Write a program in some language (python is probably a good choice, and I’ve provided some starter code, but you can choose your own) that takes as input an equation and a symbolic variable. Your program should then use an informed search procedure to simplify and attempt to solve the equation. It should focus on symbolic manipulation of the algebraic expressions, and not focus on the numerical computing. For example, floating point, exact integer (and rational numbers), and symbolic ($\pi$, $e$) values should be kept as separate.

Your assignment requires that you think about the mathematical rules and identities which can be employed as actions in a search procedure (sometimes call operators) in order to attempt to solve for a given variable. Rules to include (these are examples, and not exhaustive; you are expected to start with these and add your own embellishments), the following:

Arithmetic evaluation

Arithmetic evaluation which will allow a transformations like:

Example input:
Eq.> \( x = (2 + 10) \times (2^2) \)

Var.> \( x \)

Produces output: \( x = 48 \)

Another example input:

Eq.> \( x = 6 \times 2 / (-1 + 4 \times 0 + 1) \)

Var.> \( x \)

Produces output: \( x = \text{undefined} \)

**Applying inverses**

Simplifications involving the definition of inverse operations for standard operators should allow for solutions such as:

Example input:

Eq.> \( (2 \times \sqrt{x} \times 3) - y = \pi \)

Var.> \( x \)

Produces output: \( x = ((\pi + y)/6)^2 \)

(Or potentially other variations of this final output, depending on how far the arithmetic evaluation proceeds.)

**Associativity and Commutativity**

Several of the basic operations (e.g., addition, multiplication, etc.) are associative and commutative. Incorporating these aspects should allow simplifications as follows:

Eq.> \( (2 \times x \times 3 \times y \times 4 \times z \times 5 \times 6) = 800 \)

Var.> \( x \)

Produces output: \( x = 80 / (y \times z) \)

**Identities**

Several logarithmic identities and rules may be incorporated in your treatment of simplification by computing “inverses.” You can further supplement this with a large number of trigonometric identities that, for example, enable the following simplifications:

Eq.> \( e^x = z \times (\sin(y)^2 + \cos(y)^2) \)

Var.> \( x \)

Produces output: \( x = \log(z) \)

A similar, but rather more challenging instance is:

Eq.> \( e^x = \sin(8 + 3/2 \times z + y - 1/2 \times z)^2 + \cos(y + 8 + z)^2 \)

Var.> \( x \)

Produces output: \( x = 0 \)
Calculus

Operators for differentials and integrals allow for further simplifications:

Example input:

Eq.> Diff(x^2 + 10x + 2, x) = 4 * z
Var.> x

Produces output: x = 2 * z - 5

An extremely challenging instance (via integration by parts) is:

Eq.> x = Integrate(z * sin(z), z)
Var.> x

Produces output: x = -z * cos(z) + sin(z) + C

Code and Resources

In order to take care of the tedious input parsing component of this project, I have provided Python code for parsing an input string (in infix form), producing a parse tree, and for outputting this tree (in both infix and prefix forms). The code is available for download at:

http://robotics.cs.tamu.edu/dshell/cs625/asgn3/equationparser-0.1.tar.gz

The code makes use of the open-source PLY (Python Lex-Yacc) Library. It has been tested on GNU/Linux using Python 2.7.3, PLY 3.4. It is intended as example code, providing functionality to parse most of the input forms, with the exception of the (Diff and Integrate) examples above. You will need to extend the code in order to implement the calculus operators.

Additionally, you may find it useful to make use of the example Python code associated with the book (e.g., for A * search) available at

https://code.google.com/archive/p/aima-python/

Submission

Write a short report showing a few examples of your code being run. It should be no more than a couple of pages of the main content, but that should include:

• Your name and UIN.

• A detailed description of your approach, including:
  – A description of the search algorithm.
  – A compete list of the actions you have developed, preferably grouped by type. (It is assumed that you will not be able to do all of the ones I’ve listed!)
- A specification of the heuristic you employ, and the rationale behind it.

- Output from your program, for a variety of examples including the instances above, and other examples you cook up to illustrate the effectiveness of your program.

- A description of how to run the submission.

- Any few brief notes that might simplify the process of understanding your code.

- Affix your code as an appendix. (Not counted toward the page quota.)

- Specific notes about known bugs, issues, limitations, or errors.

- Documentation of resources used and/or help received.

Submission of the document (as a PDF) will be facilitated via the canvas site. The deadline posted on the course webpage will be the official date.