Question

Consider the following puzzle. It is in the vein of Russell and Norvig’s toy problem on page 73, that they attribute to Donald Knuth.

You are given a pair of numbers: the first is the seed, \( x_0 \), and the second the target \( x_f \).
You are provided, additionally, with two functions \( f : \mathbb{R} \to \mathbb{R} \) and \( g : \mathbb{R} \to \mathbb{R} \).

Puzzle: Write a program to give either the minimal number of function applications (of \( f \) and \( g \)) needed to reach \( x_f \) from \( x_0 \), or state that one cannot reach \( x_f \) from \( x_0 \).

Approach

This problem is from https://nmattia.com/posts/2016-07-31-bfs-tree.html
In that blog post, Nicolas Mattia considers the two functions $f$ and $g$ defined as follows:

\[ f(x) = 2x + 1 \quad \text{and} \quad g(x) = 3x + 1. \]

The author provides code to solve exactly this problem on that page. Actually, that page is code, written in Haskell. (If you’ve never heard of it: look up literate programming.)

**Task 1**

Convert his solution, for those two particular $f$ and $g$ functions, to Scheme. His approach is not quite the same as the straightforward stack-based method given in Russell and Norvig. Instead, it uses Haskell’s lazy evaluation to provide a sort of infinite data structure. You are to do the same thing: implement the search using lazy evaluation.

We have Gambit Scheme installed on the compute.cs.tamu.edu server, but you might prefer to install your own version locally so you can tinker more freely. The page at [http://www.iro.umontreal.ca/~gambit/doc/gambit.html](http://www.iro.umontreal.ca/~gambit/doc/gambit.html) provides some information that you might find useful in getting up and running with the interpreter and compiler.

By default Scheme uses eager evaluation. You have to make use of the `delay` and `force` keywords. Details on this can be found here: [http://www.shido.info/lisp/scheme_lazy_e.html](http://www.shido.info/lisp/scheme_lazy_e.html)

(Note that the text in that guide uses Guile, not Gambit Scheme, so the promise? type checking function isn’t available for us. The other functionality is supported, however.)

**Task 2**

Reflect on these things:

- Consider Knuth’s original problem. That requires more than just an $f$ and $g$. Can you pose his problem with a modification of your code?

- What properties are needed to ensure that your program is guaranteed to give a solution to the puzzle?
Submission

Write a short report showing a few examples of your code being run. It should be no more than a couple of pages of the main content, but that should include:

- Your name and UIN.
- Any few brief notes that might simplify the process of understanding your code.
- Specific notes about known bugs, issues, limitations, or errors.
- Documentation of resources used and/or help received.
- Your reflections from Task 2, and other thoughts it spurred.
- Affix your code as an appendix. (Not counted toward the page quota.)

Submission of the document (as a PDF) will be facilitated via the canvas site. The deadline posted on the course webpage will be the official date.